Int. J. Heat Mass Transfer. Vol. 26, No. 3, pp. 335–340, 1983
Printed in Great Britain

0017-9310/83/030335-06 $03.00/0
Pergamon Press Ltd.

EXERGY CONSERVATION IN PARALLEL THERMAL INSULATION SYSTEMS

W. SCHULTZ† and A. BEJAN
Department of Mechanical Engineering, University of Colorado, Boulder, CO 80309, U.S.A.

(*Received* 7 *January* 1982)

Abstract—This paper is a Second-Law study of thermal insulation systems consisting of two 1-dim. insulations in parallel. It is shown that the parallel-insulation model applies to power and refrigeration systems exposed to large absolute temperature ratios. The insulation design which conserves maximum exergy (available work) is determined for two classes of parallel-insulation systems: in one class, the two parallel insulations are cooled continuously by two streams, in the other class they are cooled partly (intermittently) by one stream of single-phase fluid. The study shows that, in contradiction to previously published results, the continuous-cooling method is thermodynamically superior to the partial-cooling method.

NOMENCLATURE

- A, cross-sectional area normal to insulation heat current Q;
- c_p, specific heat at constant pressure;
- k, effective thermal conductivity of the 1-dim. insulation;
- L, insulation thickness (length of heat current path);
- $\dot{m}$, coolant mass flowrate;
- M, dimensionless flowrate, equation (7);
- N_S, entropy generation number, equation (8);
- N_{tu}, number of heat transfer units of the counterflow heat exchanger;
- Q_C, cold-end heat current;
- Q_H, hot-end heat current;
- Q_z, local value of heat current;
- S_{gen}, entropy generation rate [W K^{-1}];
- T, absolute temperature;
- T_C, cold-end temperature;
- T_{CR}, temperature of cross-over point (Fig. 6);
- T_H, hot-end temperature;
- x, fraction of coolant flowrate (Fig. 2);
- z, position along the 1-dim. insulation (Fig. 1).

Greek symbols
- ξ, dimensionless cross-over location, equation (13).

Subscripts
- c.h.e., counterflow heat exchanger;
- min, minimum;
- opt, optimum;
- $(\ )_1$, pertaining to insulation no. 1;
- $(\ )_2$, pertaining to insulation no. 2.

INTRODUCTION

THERMAL insulation systems occupy an important role in the engineering of energy efficient systems for power and refrigeration. The importance of insulation systems is enhanced in applications which face extreme absolute temperature ratios, for example, in advanced-cycle power plants and in cryogenic gas liquefaction installations. The traditional view in the design of thermal insulation systems is that these systems' basic function is to prevent (limit) the flow of heat between the ambient and the heart of the apparatus to be insulated. An alternative, more comprehensive, way of viewing insulation systems is to recognize that they are steady dissipators of available work (exergy, availability), in other words, steady producers of entropy. This second view was proposed by one of the authors [1] who showed that the accounting for exergy destruction in insulation systems is the only avenue toward the design of truly (thermodynamically) efficient power and refrigeration systems.

Thermal insulations owe their dissipative character to the thermodynamic irreversibility associated with heat transfer across finite temperature differences [2]. Thus, in a 1-dim. insulation of effective thermal conductance kA/L (Fig. 1), the entropy generation rate is [1]

$$S_{gen} = \int_{T_C}^{T_H} (Q_z/T^2)\,dT \tag{1}$$

where T_H, T_C are the extreme temperatures and Q_z is the heat current from T_H to T_C. As shown in the present treatment, the heat current Q_z is, in general, a function of position z along the insulation. The heat current function Q_z depends on the amount of intermediate cooling provided to the insulation between $z = 0$ and $z = L$. In Fig. 1, the intermediate cooling effect is symbolized by the single-phase stream $\dot{m}c_p$ which cools the insulation in countercurrent with the heat flow Q_z. It has been shown that the destruction of exergy in the 1-dim. insulation is minimized when the coolant flowrate is [1]

$$(\dot{m}c_p)_{opt} = (Ak/L)\ln(T_H/T_C). \tag{2}$$

PARALLEL INSULATION SYSTEMS

The objective of this paper is to analyze the destruction of available work in *parallel* insulation

† Visiting from the University of Stuttgart, West Germany.

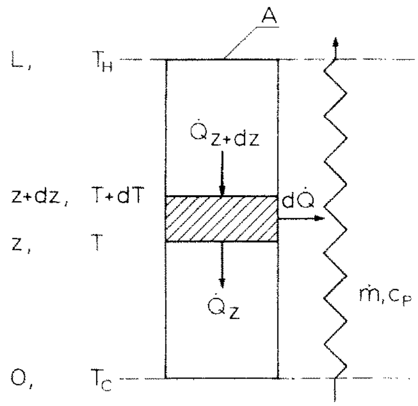

FIG. 1. Schematic of 1-dim. continuously-cooled thermal insulation system, showing internal heat transfer.

systems, as shown in Figs. 2 and 6. The parallel-insulation model recognizes the fact that in many power and refrigeration systems the insulation effect is provided by two separate installations. For example, in a helium liquefier we first distinguish an elaborate insulating layer which consists of radiation shields, evacuated space and low-heat-leak mechanical supports [3]. The second, more subtle, insulation feature is the main counterflow (regenerative) heat exchanger which connects the room temperature compressor with the low temperature end of the liquefaction process [4]. This heat exchanger leaks heat in the (hot end)–(cold end) direction, in the same sense as the 1-dim. insulation sketched in Fig. 1. It has been shown that the equivalent end-to-end thermal conductance of a balanced counter-flow heat exchanger (c.h.e.) is [1]

$$(kA/L)_{\text{c.h.e.}} = \dot{m}c_p/N_{\text{tu}} \tag{3}$$

where $\dot{m}c_p$ is the capacity rate and N_{tu} the number of heat transfer units. The end-to-end conductance decreases as the stream-to-stream conductance (or N_{tu}) increases.

Counterflow heat exchangers play a thermal insulation function not only in cryogenic systems but also in advanced power cycles (e.g. the Brayton cycle with regenerative heat exchanger [5]). The analogy between counterflow heat exchangers and traditional insulations, equation (3), justifies the parallel-insulation model employed in this study (Figs. 2 and 6). In this model, one insulation (k_1, A_1, L_1) represents the physical insulating layer (shields, vacuum, supports), while the second insulation (k_2, A_2, L_2) accounts for the role played by the counterflow heat exchanger.

In what follows we consider the engineering question of how to minimize the destruction of exergy in a parallel insulation system, by properly using one stream of cooland, $\dot{m}c_p$. In the first part of the study we focus on the arrangement shown in Fig. 2, where fractions of $\dot{m}c_p$ are in continuous thermal contact with each of the two insulations. In the second part we consider the method of Fig. 6 in which the available stream ($\dot{m}c_p$) cools both insulations without being fractioned into two substreams. For both methods we determine the optimum operating conditions which guarantee the destruction of least exergy. We then compare the relative merits of the two methods; the purpose of this discussion is to correct previously published results [6] which wrongly suggest that the discrete cooling of parallel insulations is thermodynamically superior to continuous cooling.

TWO-STREAM CONTINUOUS COOLING

The insulation system considered in this section is shown in Fig. 2. The two insulations are positioned between the same extreme temperatures (T_H, T_C), and are being cooled continuously by fractions x and $(1-x)$ of a total gas stream $\dot{m}c_p$. Applying the Second Law of Thermodynamics to the two cross-hatched areas which house the thermodynamic irreversibility, we obtain the rate of entropy generation for the entire system.

$$S_{\text{gen}} = \int_0^{L_1} (\mathrm{d}Q/T) + (Q_{C1}/T_C) - (Q_{H1}/T_H)$$
$$+ \int_0^{L_2} (\mathrm{d}Q/T) + (Q_{C2}/T_C) - (Q_{H2}/T_H). \tag{4}$$

The heat current notation employed in equation (4) is defined clearly on Fig. 2. The two streams $x\dot{m}c_p$ and $(1-x)\dot{m}c_p$ are in local thermal equilibrium with their respective insulations. Invoking the relationship between heat current, thermal conductance and local temperature gradient,

$$Q_z = (kA/L)\frac{\mathrm{d}T}{\mathrm{d}(z/L)} \tag{5}$$

the entropy generation rate can be written in terms of geometric parameters. Omitting the ensuing algebra, the final expression is

$$S_{\text{gen}} = \dot{m}c_p[(T_H/T_C)-1]$$
$$\times [\{[1-(T_C/T_H)e^{M_1x}]x/(e^{M_1x}-1)\}$$
$$+\{[1-(T_C/T_H)e^{M_2(1-x)}](1-x)/(e^{M_2(1-x)}-1)\}]$$
$$+\dot{m}c_p \ln(T_H/T_C). \tag{6}$$

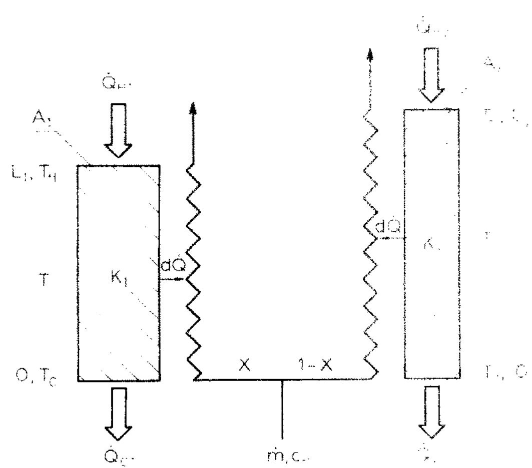

FIG. 2. Schematic of parallel-insulation system cooled continuously by two parallel streams.

In this expression, M_1 and M_2 are the dimensionless capacity rates corresponding to the two insulations,

$$M_1 = (\dot{m}c_pL_1)/(k_1A_1), \quad M_2 = (\dot{m}c_pL_2)/(k_2A_2). \quad (7)$$

It is convenient to express the entropy generation rate in dimensionless form also, by defining the entropy generation number N_S

$$N_S = (S_{\text{gen}}L_1/k_1A_1) = M_1[(T_H/T_C)-1] \times [\{[1-(T_C/T_H)e^{M_1x}]x/(e^{M_1x}-1)\} + \{[1-(T_C/T_H)e^{M_2(1-x)}](1-x)/(e^{M_2(1-x)}-1)\}] + M_1 \ln(T_H/T_C). \quad (8)$$

The N_S expression (8) is the object of minimization. For a given absolute temperature ratio T_H/T_C and a given conductance ratio

$$M_1/M_2 = (kA/L)_2/(kA/L)_1, \quad (9)$$

we must optimally select the two remaining parameters, the flow fraction x and the mass flowrate M_1 (or M_2). The standard analytical method of setting equal to zero the partial derivatives, $\partial N_S/\partial x$ and $\partial N_S/\partial M_1$, yields two implicit equations. Since the solution to this system requires about the same numerical effort as searching for the N_S minimum directly, we minimized N_S by trial and error.

The results of our N_S minimization work are reported for T_H/T_C values of 10 and 100, which represent very well helium-cooled insulations. We varied the conductance ratio M_1/M_2 from 0.1 to 10, and determined the optimum x and M_1 for minimum N_S. Figures 3 and 4 show the minimum entropy generation number $N_{S,\min}$ and the optimum flowrate number $M_{1,\text{opt}}$ as functions of the insulation conductance ratio M_1/M_2. Both N_S and M_1 scale with the thermal conductance of insulation no. 2. Since higher values of M_1/M_2 correspond to more heat leaking into the system, $N_{S,\min}$ and $M_{1,\text{opt}}$ increase as M_1/M_2 increases. Increasing the absolute temperature ratio T_H/T_C has the same effect. However, increasing this ratio from 10 to 100 leads to an increase in $N_{S,\min}$ by only a factor of order 4 (instead of by a factor of 10 as in the case of insulations without lateral stream-cooling effect).

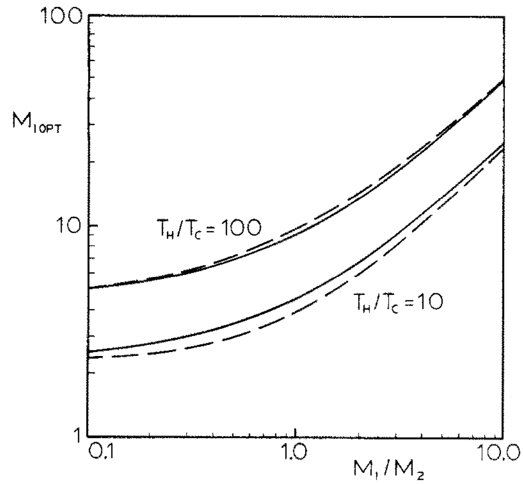

FIG. 4. Optimum mass flowrates for continuous cooling (solid lines) and intermittent cooling (dashed lines) of parallel-insulation systems.

The optimum flow fraction x was found to vary antisymmetrically with respect to $\log(M_1/M_2) = 0$. This result has been plotted in Fig. 5. As we might have expected, the optimum fraction is $x = 0.5$ when parallel insulations are identical ($M_1/M_2 = 1$). When the two insulations differ drastically, the one with the higher thermal conductance demands most of the coolant supply. It can be shown analytically (based on $\partial N_S/\partial x = \partial N_S/\partial M_1 = 0$) that the optimum x is also a function of the temperature ratio T_H/T_C. This dependence, however, is too weak to become visible in Fig. 5.

SINGLE-STREAM INTERMITTENT COOLING

A parallel-insulation system of superior technical simplicity is shown in Fig. 6. This system employs only

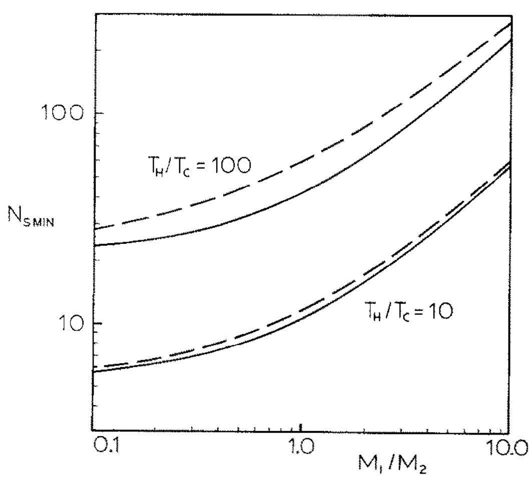

FIG. 3. Minimum entropy generation numbers for continuous cooling (solid lines) and intermittent cooling (dashed lines) of parallel-insulation systems.

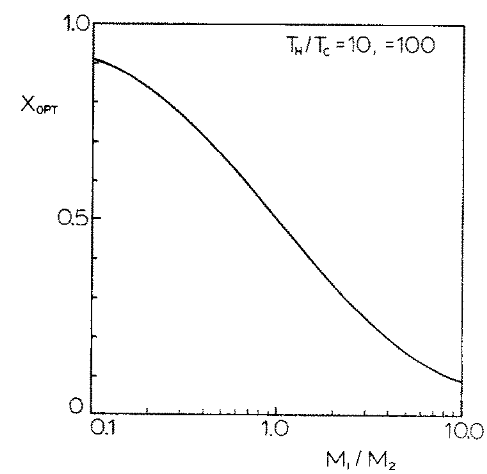

FIG. 5. Optimum flow fraction vs support conductance ratio for parallel continuous cooling scheme.

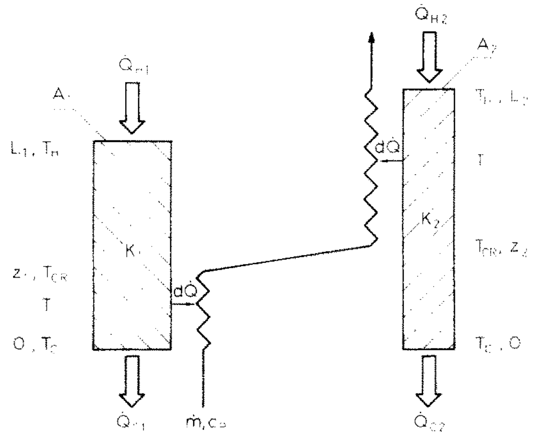

FIG. 6. Schematic of parallel insulation system partly cooled by one stream.

one stream ($\dot{m}c_p$) which comes in contact with the lower part of insulation no. 1 up to a thickness $z = z_1$, and with the upper part of insulation no. 2 from $z = z_2$ to $z = L_2$. The stream temperature at the cross-over point is T_{CR}. The stream comes in contact with insulation no. 2 at the point where the insulation temperature is equal to T_{CR}: this cross-over design is the most reasonable (least irreversible).

The total rate of entropy generation of this system is,

$$S_{gen} = \int_0^{z_1} (dQ/T) + (Q_{C1}/T_C) - (Q_{H1}/T_H) + \int_{z_2}^{L_2} (dQ/T) + (Q_{C2}/T_C) - (Q_{H2}/T_H) \quad (10)$$

where the heat current terminology has been defined in Fig. 6. It can easily be shown that the heat currents at the stream-cooled ends of the two insulations are given by

$$Q_{C1} = \dot{m}c_p T_C\{[(T_{CR}/T_C)-1]/(e^{M_1\xi_1}-1)\}, \quad (11)$$

$$Q_{H2} = \dot{m}c_p T_{CR}\{[(T_H/T_{CR})-1]/(e^{M_2(1-\xi_2)}-1)\}e^{M_2(1-\xi_2)} \quad (12)$$

where

$$\xi_1 = z_1/L_1, \quad \xi_2 = z_2/L_2, \quad (13)$$

are the dimensionless cross-over locations. As in the preceding section, M_1 and M_2 are the mass flow numbers defined in equation (7). The heat currents at the two uncooled ends of the insulations are given by

$$Q_{H1} = T_H(k_1A_1/L_1)\{[1-(T_{CR}/T_H)]/(1-\xi_1)\}, \quad (14)$$

$$Q_{C2} = T_C(k_2A_2/L_2)\{[(T_{CR}/T_C)-1]/\xi_2\}. \quad (15)$$

Introducing equations (11)–(15) into equation (10), and performing the integrals in equation (10), yields, after some algebra,

$$\begin{aligned} N_S = {} & M_1[(T_{CR}/T_C)-1] \\ & \times \{[1/(e^{M_1\xi_1}-1)]+[1/(M_2\xi_2)]\} \\ & + M_1[(T_{CR}/T_H)-1] \\ & \times \{[1/(1-e^{-M_2(1-\xi_2)})]+1/[M_1(1-\xi_1)]\} \\ & + M_1 \ln(T_H/T_C). \end{aligned} \quad (16)$$

The entropy generation number N_S is defined as in the preceding section [equation (8)].

The cross-over temperature T_{CR} appearing in equation (16) may be eliminated based on the following condition of heat current continuity. Considering insulation no. 1, the constant heat current through the uncooled part, Q_{H1}, must be equal to the heat current into the upper end of its cooled part. Therefore, we can write

$$T_H(k_1A_1/L_1)\{[1-(T_{CR}/T_H)]/(1-\xi_1)\} = T_C\dot{m}c_p\{[(T_{CR}/T_C)-1]/(e^{M_1\xi_1}-1)\}e^{M_1\xi_1} \quad (17)$$

or

$$T_{CR}/T_C = [M_1(1-\xi_1)+(T_H/T_C)(1-e^{-M_1\xi_1})] \div [M_1(1-\xi_1)+1-e^{-M_1\xi_1}]. \quad (18)$$

The same condition applied to insulation no. 2 yields

$$T_{CR}/T_C = [e^{M_2(1-\xi_2)}+(T_H/T_C)M_2\xi_2-1] \div [e^{M_2(1-\xi_2)}+M_2\xi_2-1]. \quad (19)$$

Eliminating T_{CR}/T_C between equations (18) and (19) leads to an implicit relationship between the cross-over locations of the two insulations

$$\xi_2 = (1-e^{-M_1\xi_1})[e^{M_2(1-\xi_2)}-1]/[M_1M_2(1-\xi_1)]. \quad (20)$$

The temperature ratio T_H/T_C and the conductance ratio M_1/M_2 are known. Consequently, equations (16), (18) and (20) enable us to calculate the entropy generation number as a function of two independent parameters ξ_1 and M_1. The minimum entropy generation number was determined numerically by trial and error. Figures 3 and 4 show $N_{S,min}$ and $M_{1,opt}$ as functions of M_1/M_2 and T_H/T_C. As in the system of Fig. 2, higher values of M_1/M_2 and T_H/T_C lead to higher values of $N_{S,min}$ and $M_{1,opt}$. The optimum cross-over locations ξ_1 and ξ_2 are reported in Figs. 7 and 8: in these

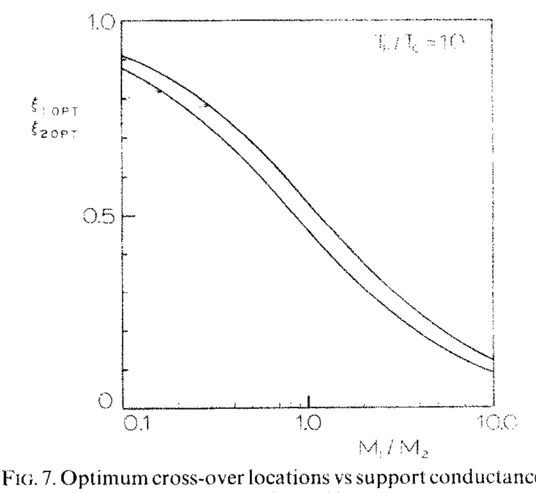

FIG. 7. Optimum cross-over locations vs support conductance ratio ($T_H/T_C = 10$).

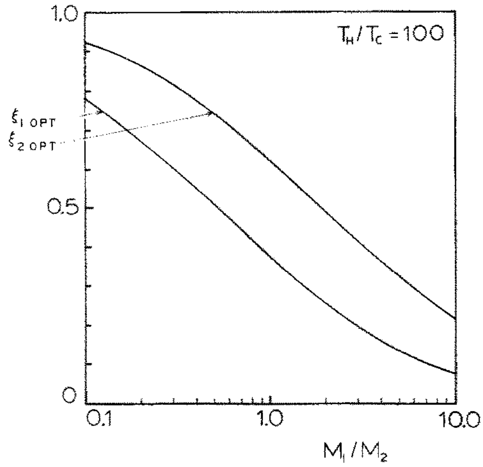

FIG. 8. Optimum cross-over locations vs support conductance ratio ($T_H/T_C = 100$).

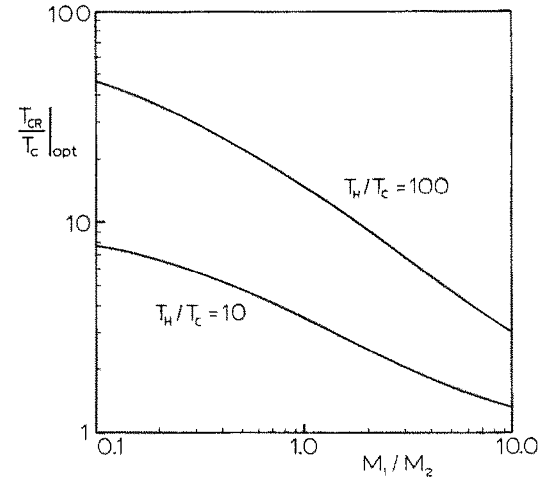

FIG. 9. Optimum cross-over temperature vs support conductance ratio.

figures the arithmetic mean of ξ_1 and ξ_2 shows an antisymmetric variation relative to $\log(M_1/M_2) = 0$. If insulation no. 2 houses the dominant heat leak, i.e. if $M_2 \ll M_1$, then it must be cooled over most of its length. The reduced cross-over temperature T_{CR}/T_C is shown in Fig. 9 as a function of M_1/M_2 and T_H/T_C.

In contrast to the fractioned-flow system considered earlier, the temperature ratio T_H/T_C has a sizeable effect. Figures 7 and 8 show that the greater the ratio T_H/T_C, the greater the displacement between cross-over locations, $\xi_2 - \xi_1$. Note that the difference $\xi_2 - \xi_1$ is always positive, because the upper-section temperature gradient is greater in a stream-cooled section than in an uncooled section. This effect is shown in Fig. 10, which is a qualitative sketch of the temperature distribution in two parallel insulations of the same length.

DISCUSSION

We are now in a position to assess the relative thermodynamic merit of the parallel-insulation designs proposed in Figs. 2 and 6. Both designs have been optimized thermodynamically by determining the operating conditions which insure minimum destruction of exergy in the given system. Figure 3 shows that the single-stream system (Fig. 6) is thermodynamically inferior to the two-stream system (Fig. 2). The difference between the minimum entropy generation numbers increases as the temperature ratio T_H/T_C increases. Therefore, the two-stream arrangement is particularly desirable in helium cryogenic systems, where the temperature ratio is of order 100.

The optimum cooling requirements for the two cooling methods are summarized in Fig. 4. This summary shows that in the $(T_H/T_L)-(M_1/M_2)$ range considered in our study, the optimum total flowrate is relatively insensitive to the way in which the flow is ducted. Of special interest is the fact that when $T_H/T_C = 100$ the two-stream continuous cooling method requires *less* coolant than the one-stream method (Fig. 14). In a helium refrigeration cycle, the coolant stream $\dot{m}c_p$ is bled from the low temperature end of the cycle and is ducted toward cooling both the insulating jacket and the main counterflow heat exchanger. The continuous cooling of the main counterflow heat exchanger is effected by augmenting the flowrate on the low pressure side relative to the flowrate on the high pressure side of the heat exchanger [7].

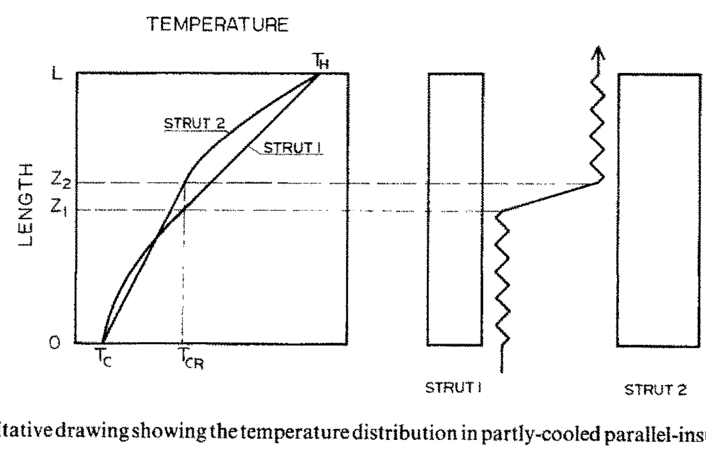

FIG. 10. Qualitative drawing showing the temperature distribution in partly-cooled parallel-insulation system.

The fundamental result of our study is the fact that for maximum exergy conservation parallel-insulation systems must be cooled *continuously*, as in Fig. 2. Despite their relative simplicity, intermittent cooling arrangements of the type shown in Fig. 6 cannot match the thermodynamic performance of the continuous-cooling arrangement. This conclusion invalidates the claim made recently by Hilal and Eyssa [6], who considered the thermodynamic optimization (exergy conservation design) of a large scale cryogenic system. These authors addressed the same question as in our study, namely, that of the best allocation and ducting of cold fluid $\dot{m}c_p$ between the two parallel insulations of the system (1. the insulation jacket, and 2. the main counterflow heat exchanger column). Hilal and Eyssa [6] concluded that, compared with the continuous cooling method, the discrete (intermittent) cooling method yields greater savings in refrigerator power (exergy). Their conclusion is incorrect: the error stems from the fact that in comparing various cooling methods, they did not constrain the parallel-insulation system. Specifically, the N_{tu} of the main heat exchanger (hence, its end-to-end thermal conductance) varied from one cooling method to another. Hilal and Eyssa [6] constrained only one leg of the parallel-insulation system, namely, the insulating jacket.

REFERENCES

1. A. Bejan, A general variational principle for thermal insulation system design, *Int. J. Heat Mass Transfer* **22**, 219 (1979).
2. H. D. Baehr, *Thermodynamik*, pp. 128–156. Springer, Berlin (1973).
3. R. H. Kropschot, B. W. Birmingham and D. B. Mann (editors), *Technology of Liquid Helium*. N.B.S. Monograph 111 (1968).
4. S. Foner and B. B. Schwartz (editors), *Superconducting Machines and Devices*, Chapter 5. Plenum Press, New York (1974).
5. R. W. Haywood, *Analysis of Engineering Cycles*, Chapter 6. Pergamon Press, Oxford (1967).
6. M. A. Hilal and Y. M. Eyssa, Minimization of refrigeration power for large cryogenic systems, in *Advances in Cryogenic Engineering*, Vol. 25, p. 350. Plenum Press, New York (1980).
7. R. H. Hubbell and W. M. Toscano, Thermodynamic optimization of helium liquefaction cycles, in *Advances in Cryogenic Engineering*, Vol. 25, p. 551. Plenum Press, New York (1980).

ECONOMIE D'ENERGIE DANS LES SYSTEMES EN PARALLELE D'ISOLATION THERMIQUE

Résumé—On étudie par la seconde loi les systèmes d'isolation thermique composé de deux isolations monodimensionnelles en parallèle. On montre que le modèle s'applique aux systèmes de puissance et de réfrigération exposés à de grands rapports de températures absolues. Le système qui conserve l'énergie au maximun (travail disponible) est déterminé pour deux classes de systèmes : dans une classe, les deux isolations en parallèle sont refroidies continuement par deux écoulements, dans l'autre elles sont refroidies partiellement (de façon intermittente) par un écoulement de fluide monophasique. L'étude montre que, contrairement à des résultats précédemment publiés, la méthode de refroidissement continu est thermodynamiquement supérieure à la méthode de refroidissement partiel.

EXERGETISCHE OPTIMIERUNG DER FLUIDKÜHLUNG PARALLELER WÄRMELEITER

Zusammenfassung—Mit Hilfe des zweiten Hauptsatzes der Thermodynamik wird der Exergieverlust in zwei parallelen fluidgekühlten Wärmeleitern untersucht. Es wird gezeigt, daß das Modell paralleler eindimensionaler Wärmeleiter auf Wärmekraft- und Kälteprozesse anwendbar ist. Die Betriebsbedingungen des Fluid-Wärmeleiter-Systems mit dem geringsten Verlust an Exergie (technisch verfügbarer Arbeit) werden für zwei Kühlkonzepte ermittelt : 1. Beide Wärmeleiter werden von zwei parallelen Strömen (Ein-Phasen-Fluid) kontinuierlich gekühlt. 2. Sie werden von einem (überwechselnden) Strom je nur zum Teil gekühlt. Die Arbeit kommt zu dem Schluß, daß entgegen unlängst hierzu veröffentlichten Ergebnissen durch kontinuierliche Kühlung größere Exergieeinsparungen als durch teilweise (aussetzende) Kühlung zu erreichen sind.

СОХРАНЕНИЕ ЭНЕРГИИ В СИСТЕМАХ ПАРАЛЛЕЛЬНОЙ ТЕПЛОВОЙ ИЗОЛЯЦИИ

Аннотация—В настоящей работе проведено исследование одномерных двухслойных систем тепловой изоляции с учетом второго закона термодинамики. Показано, что модель параллельной изоляции применима к энергетическим и морозильным системам при больших абсолютных перепадах температур. Конструкция изоляции, сохраняющей максимальную энергию (полезная работа), определена для двух классов систем параллельной изоляции: в системах первого класса две параллельных изоляции непрерывно охлаждаются двумя потоками, в системах другого класса они охлаждаются раздельно (попеременно) одним потоком однофазной жидкости. Исследования показывают, что, вопреки ранее опубликованным результатам, метод непрерывного охлаждения с точки зрения термодинамики выгоднее, чем метод попеременного охлаждения.